

ADVANCED MACROECONOMICS II

Lecture Notes I

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Modern macroeconomics...

... is built on microfoundations

What does it mean? It means that in order to build a realistic model one must solve for optimal decisions of all agents (households, firms and government) in a general equilibrium setting.

Static models that assume structural relationships between variables (for example, the dependence between current and permanent income) may lead to erroneous conclusions.

This is a basic idea behind the Lucas (1976) critique that changed the way of how macroeconomists think.

A snapshot of a typical macro model:

1. Households have a utility function and choose their consumption, labor supply and investment given their budget constraints
2. Firms choose labor and capital to maximize profits given their technology and prices
3. Government chooses taxes to finance social security or public goods

In the end, optimal decisions depend on the parameters in the model

The decisions of three types of economic agents are aggregated (in general equilibrium) to solve for optimal prices and aggregate quantities

Extensions may include:

- more than one type of agents (for example, multiple generations in overlapping generations models - OLG)
- agents may choose more than one type of consumption each period (durables and non-durables)
- agents may be constrained in their borrowing
- uncertainty in firm's production (for example, stochastic output due to weather conditions)
- time may be finite or infinite (depending on whether agents care about their offsprings)

and so on

The simplest dynamic economy:

A two-period endowment economy: a farmer receives endowment of wheat y_1 and y_2 in periods 1 and 2

For each kilo of wheat she plants in period 1 she will receive $(1+r)$ kilos next period

Model setup

Question: Which variables in the budget constraint are exogenous? (What is "exogenous" variable?)

Suppose that agent's utility is given by

Question: Is this model stochastic?

Question: Is it a general equilibrium model?

Solving the model

First Order Conditions

Model Equilibrium

Consumption in the second period becomes

and in the first period

Implications of the first dynamic economy

Question: What will happen to c_2 if r goes up?

Question: What will happen to c_1 if β goes up?

In either case (change in the exogenous parameter r or β) adjusts **both** consumption choices: c_1 and c_2 . Hence, we talk about **dynamic equilibrium models**.

How can we endogenize the interest rate?

One way is to introduce 2 types of agents with different endowments in 2 periods.

In order to smooth consumption one type will save and another type will borrow in the first period.

In the second period the borrowers will pay back $(1+r)$ for each unit they borrow. In the end, we can solve for the **endogenous** interest rate that comes from the model equilibrium.

Recall Homework 3 from Macro I - your first economy with heterogeneous agents-:)

A more realistic model with PRODUCTION

Consider a household where a wife has some capital she can give to her husband to invest in business

Assume that they have joint utility as a household (i.e. eat at one table) without separate (individual) utilities

Alternative: Robinson Crusoe economy

Setup of the model with Production

The household maximizes

Suppose that $y_1 = y$, $y_2 = 0$ and the amount k saved in the first period can be used to invest in the second period according to $f(k) = k^\alpha$.

Question: How can we modify the utility above in terms of k and exogenous parameters?

Solving the model

Model equilibrium

Adding Uncertainty

Suppose that in our production economy the payoff in year 2 can take 2 values depending on the weather.

For example, in the "good" year the crop is great and in the "bad" year the crop is poor.

Our production function will now look like

Expected Utility

Question: Does shock s depend on the shock in the previous year (i.e. is there a serial correlation)?

Suppose that agents now act as **expected utility maximizers**. (Welcome to the world of von Neumann and Morgenstern! For advanced treatment see chapter 6 in MacColell, Whinston and Green.)

Solving the stochastic model

MODEL WITH (IN)FINITELY MANY PERIODS

Finite-horizon Ramsey model

Before we consider the case of infinitely many periods let's start with the finite-horizon model.

In 1928 Frank Ramsey posed a question: "How much of its income should a nation save?" and developed a dynamic model to answer this question.

Quote from the textbook (Heer and Maussner 2009): "At the heart of the Ramsey problem there is an economic agent producing output from labor and capital who must decide how to split production between consumption and capital accumulation."

For example, a farmer eats C amount of wheat and saves K amount to plant it. Alternatively, you can think of a family deciding on how much to consume and how much to invest in the stock market.

A simpler model?

Notice: This is not the simplest model with many periods because it includes **production**.

"A Cake Eating Problem" of Gale (1967) is a simpler alternative. In this model, agent at time zero is given a fixed amount of cake that does not depreciate. The cake can be used (i) to consume today or (ii) to save and consume more tomorrow.

The question becomes: what is the optimal consumption path $\{c_t\}_{t=1}^T$ over time $t = 1, 2, \dots, T$?

Although the Ramsey model is somewhat more complicated, I chose it to save time later.

Utility

Time is measured discretely with periods denoted as $t = 0, 1, \dots, T$. There is no date $T+1$ for the farmer -:(

The farmer has to decide how much wheat to consume (C_t) and how much wheat to invest (K_{t+1}) in each period t subject to the total output:

Question: Is this constraint likely to be binding?

Farmer's utility function is

Question: Is there a labor-leisure choice in this formulation?

Capital does not fully depreciate so we define the production function as

The finite-horizon Ramsey problem...

... can now be defined as

Question: Is there any uncertainty in this formulation?

If the utility function U and the production function f are (i) strictly concave, (ii) strictly increasing, and (iii) twice continuously differentiable then the **Kuhn-Tacker Theorem** applies.

After you apply the Kuhn-Tacker theorem you will arrive at the following conditions characterizing optimal solution

Interpreting the FOCs

The Left-Hand Side (LHS) of

The Right-Hand Side (RHS) of

NUMERICAL EXAMPLE OF THE FINITE-HORIZON RAMSEY PROBLEM

For simplicity (and reasons that become clear in the infinite-horizon Ramsey model) we choose the time-separable (TAS) utility function

and the production function

Solution

Using the optimal conditions from above

Solution II

Question: Suppose $T = 4$. What variables enter into equation for each $t = 0, 1, \dots, T-1$?

$$t = 0$$

$$t = 1$$

$$t = 2$$

$$t = 3$$

Solution III

Hence, we obtain a system of T nonlinear equations in the T unknown capital stocks K_1, K_2, \dots, K_T .

To solve this system of nonlinear equations one can use various solvers such as Newton-Raphson method.

We will not consider those methods in here - this is a topic for a course in Computational Economics (offered in the 2nd year of the program).

4 examples are presented in the attached Figure 3.1 from the Heer-Maussner textbook.