ADVANCED MACROECONOMICS II

Lecture Notes II

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Finite or Infinite Horizon?

The **finite-horizon** Ramsey model consists of T nonlinear equations that (at least in principle) can be solved by a multivariate solver such as Newton-Raphson

However, with the **infinite-horizon** Ramsey model this strategy will not work

We will need additional assumptions and new methods to handle this problem

Time Additive Separable (TAS) utility...

... can be defined as

Since economies usually do not have the terminal date T we can iterate on the TAS utility to arrive at

The Ramsey problem with infinite time horizon is then

Restrictions on Resources

Solving the model

There are two ways to characterize the solution to the infinite-horizon Ramsey model:

In the first case we maximize the following Lagrangian

Assumptions

The Farmer hates starving to death

$$\lim_{C_t \to 0} u'(C_t) = \infty \tag{1}$$

FIRST APPROACH

Two additional assumptions are needed

i) K_0 given

ii) Transversality condition:

NUMERICAL EXAMPLE OF THE INFINITE-HORIZON RAMSEY MODEL

Substituting the resource constraint $C_t = f(K_t) - K_{t+1}$ into the Euler equation obtain the system of difference equations

SECOND APPROACH - DYNAMIC PROGRAMMING

DYNAMIC PROGRAMMING II

With TAS utility consumption at t depends on K_t but not on C_{t-1}

Stock of capital at time t is the **only** necessary variable needed to determine the optimal sequence of capital stocks

Properties of the value function

Given that both $u(C)$ and $f(K)$ are strictly increasing, strictly concave and twice continuously differentiable functions of their respective arguments C and K, and that there exists a maximum sustainable capital stock, one can prove the following results:

NUMERICAL EXAMPLE

The notion of the steady state

NOTES

Homework 1 (HW1)

DUE: November 18th, in class

Homeworks submitted after that time will lose 10% of score. No homeworks will be accepted after 5 pm of the due day.

HW1: Question 1a
State the Kuhn-Tucker theorem for the non-linear programming problem above (the finite-horizon Ramsey problem)

HW1: Question 1b
Apply the Kuhn-Tucker theorem to obtain the first order conditions. Under what condition(s) on the utility function will the resource constraint be binding? What is the intuitive explanation (in terms of wheat consumption by the farmer)?

Homework 1

Question 2: Consider Example 1.2.1 on page 15 in the Heer-Maussner textbook (2nd edition). Take the specific functional forms and show that Dynamic Programming can be used to obtain the Euler equation. *Hint: Follow the discussion after equation (1.15-:)*

NEXT LECTURE: REAL BUSINESS CYCLES

The following discussion is based on *Economic Growth and Business Cycles* by Cooley and Prescott (1995)

The infinite horizon Ramsey model is the working horse of modern macroeconomics

For example, we can easily turn it into exogenous growth model

Similar models (with properly chosen preferences and production) will converge to a balanced growth path and replicate Kaldor's stylized growth facts

Re-consider Solow exercise

Moreover, we can use this new model to study the determinants of long run growth similar to Solow

After taking logs we obtain the econometric specification

After repeating Solow exercise with recent data one can obtain the following results for the US economy

Growth vs Business Cycles II

Notice that *economic growth* and *business cycles* are **not distinct** phenomena

Question: How should we modify the growth model so that it generates business cycles?

New features

Integrating both *growth* and *fluctuations* requires some features not present in deterministic growth model

A stochastic growth economy with labor

Households' utility is defined over stochastic sequences of consumption and leisure

Production

Question: Where does uncertainty come from?

The law of motion for the random productivity parameter z

Firm's problem

Aggregate capital stock evolves according to

With CRS - constant returns to scale - we can consider a single firm choosing K_t and H_t to maximize profits

Firms FOCs

After taking the first order conditions

Question: What is firm's profit given CRS production function?

Question: Is the firm's problem intertemporal?

A specific model economy

$$\max E\left[\sum_{t=0}^{\infty} \beta^t (1+\eta)^t [(1-\alpha)\log c_t + \alpha \log(1-h_t)]\right]$$

$$\begin{aligned} \text{s.t.} \quad & c_t + x_t = e^{z_t}(1-\gamma)^{t(1-\theta)} k_t^{\theta} h_t^{1-\theta}, \\ & (1+\gamma)(1+\eta)k_{t+1} = (1-\delta)k_t + x_t, \\ & z_{t+1} = \rho z_t + \epsilon_t. \end{aligned}$$

Introducing population growth

The idea behind 'calibration'

In the specific model economy there are parameters (such as θ, α etc) that have corresponding counterparts in the real economy

For example, θ is the capital share in output

This process of assigning specific values from real economy to theoretical parameters is called *calibration*

Defining business cycles

In most industrialized countries output tends to fluctuate about a long-term growth path

These fluctuations may be of varying amplitude and duration

We would like our artificial economy to replicate these fluctuations

Separating growth from business cycles

How can we divide time series of output y_t into the *growth* and the *cyclical* components?

Hodrick-Prescott filter is often used for this purpose

Successes and failures of the RBC model