ADVANCED MACROECONOMICS II

Lecture Notes III

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THIS LECTURE

✓ **Restuccia and Urrutia (2001)** - an interesting application of a standard neoclassical growth model to the study of investment rate variation around the world

✓ **Hansen (1985)** - this is one of the first models with heterogenous agents in which the markets are still *complete*

Question: How do we model market incompleteness?

Restuccia and Urrutia (2001)

Restuccia, Diego and Carlos Urrutia. 2001. Relative prices and investment rates. *Journal of Monetary Economics* Vol. 47, 93-121.

RU 2001 paper helps to understand that the basic representative agent growth model can still be used in many applications

Mankiw et al. (1992) report that in the steady state of a standard Solow model differences in investment rates explain about half of the income disparity across countries

Thus, we need to understand why investment rates differ across countries

Domestic vs international prices

Heston and Summers (1988, 1996) emphasize that investment rate differences are large when measured at a common set of prices, while very small at domestic prices

Example:

Relative prices

RU 2001: "Easterly (1993) and Jones (1994) report that the relative price of investment is higher in poor countries."

RU 2001: "This observation is consistent with the fact that rich and poor countries devote a similar fraction of income to investment expenditures, even though rich countries allocate more resources to investment."

This relative price differences may be a result of "a broad range of economic policies and institutions such as fiscal and trade policies, barriers, prohibitions, corruption, obstacles to production..." (RU 2001).

Data

This paper uses a panel of the relative price of aggregate investment for 125 countries from 1960 to 1985 from the Penn World Table (PWT)

PWT provides internationally comparable price levels of investment (PI) and consumption goods (PC) across countries using prices from International Comparisons Program

Discussion

The model

The production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{1}$$

The model II

The budget constraint of a representative household

$$C_t + (1 + \theta_t)X_t = w_t L_t + r_t K_t + T_t \tag{2}$$

where θ_t is the *tax* that household pay for each unit of investment and T_t is a lump-sum transfer.

The law of motion for capital is

$$K_{t+1} = (1 - \delta)K_t + X_t \tag{3}$$

Recursive competitive equilibrium

- Given $w(\kappa)$, $r(\kappa)$, $\tau(\kappa)$ and the aggregate law of motion $\Psi(\kappa)$, the value function $v(k,\kappa)$ solves the household's functional equation

$$v(k,\kappa) = \max_{c,x,k'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \tilde{\beta} v(k', \kappa') \right\}$$

s.t. $c + (1+\theta)x = w(\kappa) + r(\kappa)k + \tau(\kappa)$,

$(1+g)(1+n)k' = (1-\delta)k + x$,

$\kappa' = \Psi(\kappa)$,

$c, x \geq 0$,

where $g^c(k,\kappa)$, $g^x(k,\kappa)$, $g^{k'}(k,\kappa)$ are optimal decision rules for this problem;

Steady state investment rate

We obtain the following expression for the steady state investment rate:

$$\frac{x}{y} = \frac{\alpha[(1+g)(1+n)-(1-\delta)]}{(1+\theta)[1/\tilde{\beta}(1+g)(1+n)-(1-\delta)]} \tag{9}$$

depending on the exogenous growth rates of technology and population, the discount factor, technology parameters, depreciation rate, and the level of barriers to investment θ.

Notice that θ_t^j is **the only** parameter that changes across countries *j*.

Steady state

RU 2001: "To make international comparisons, we assume that each country is a closed economy, with the same depreciation rates, preferences, technology, and population growth. Then (9) can be written relative to the U.S. level as follows:"

RU 2001: "It shows that in steady state, countries with higher barriers to investment have lower investment rates. Note this relationship does not depend on α."

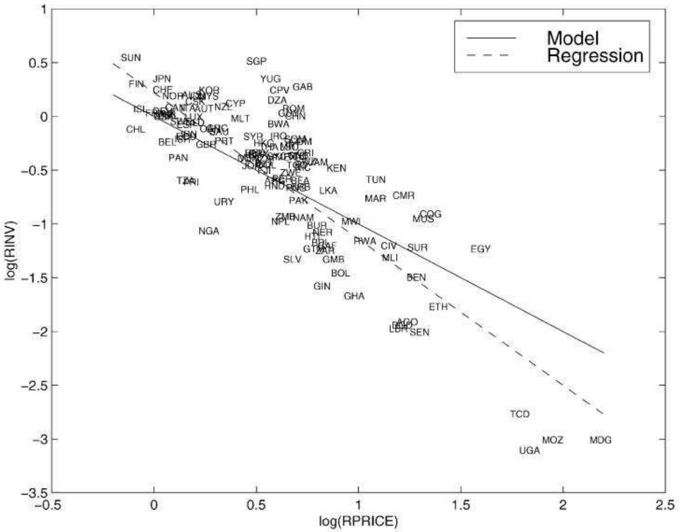

Fig. 3. Relative prices and investment rates.

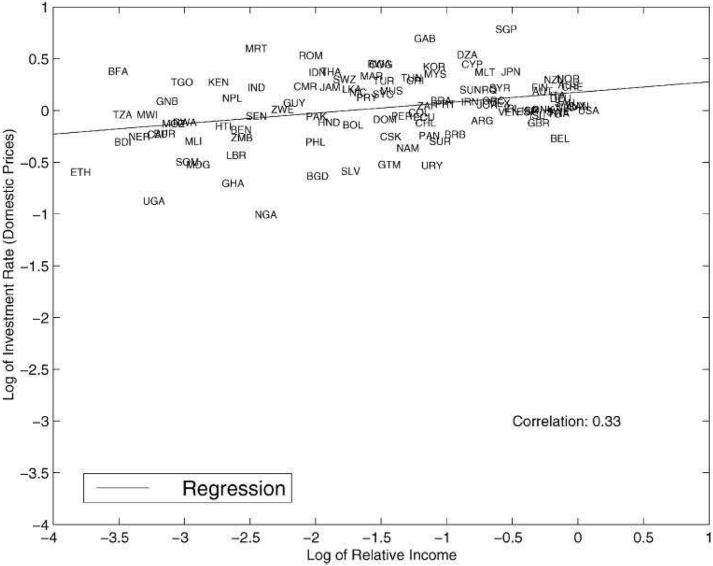

Fig. 4. Relative investment rate at domestic prices and relative income: 1985.

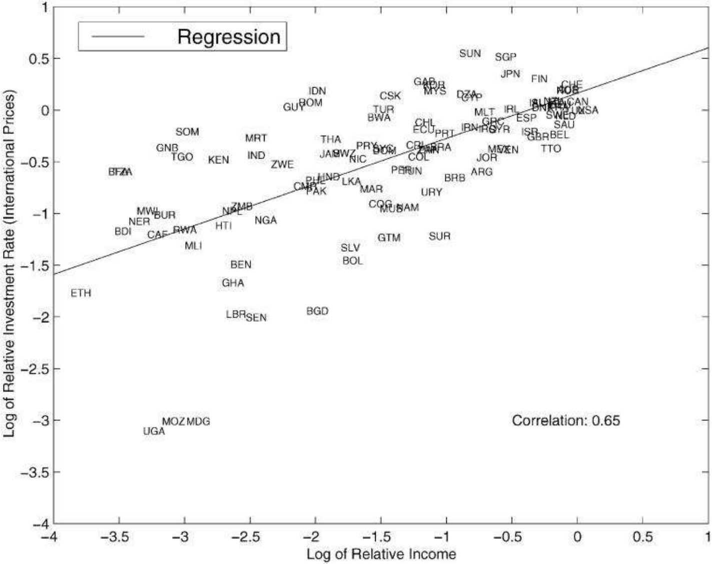

Fig. 5. Relative investment rate at international prices and relative income: 1985.

Discussion

Hansen (1985)

Hansen, Gary D. 1985. Indivisible Labor and the Business Cycle. *Journal of Monetary Economics.* Vol. 16. pp, 309-327.

This is a model in which there are two types of agents: those who "won" the employment lottery and those who did not

For computational simplicity we can construct a single "representative" agent who will be different from either type

Thus, Hansen (1985) offers a model with heterogeneous agents and complete markets that can be solved as a standard representative agent model

Motivation

Earlier equilibrium models of business cycle [Lucas (1977) and Kydland and Prescott (1982)] fail to account for some labor market phenomena (such as large fluctuations in hours relative to changes in productivity observed in the data)

Micro studies show much lower level of intertemporal substitution than that needed to explain large fluctuation in aggregate hours worked

Non-convexity is introduced in Hansen 1985 by means of indivisible labor - people either work full time or do not work at all

Hansen (1985):

"Existing equilibrium models have also failed to account for large fluctuations in hours worked along with relatively small fluctuations in productivity (or the real wage).

Prescott (1983), for example, finds that for quarterly U.S. time series, hours worked fluctuates about twice as much (in percentage terms) as productivity.

In this paper it is shown that an economy with indivisible labor exhibits very large fluctuations in hours worked relative to productivity.

This stands in marked contrast to an otherwise identical economy that lacks this non-convexity. In this economy hours worked fluctuates about the same amount as productivity."

Indivisible labor assumption...

... is supported by the data. Consider the variance decomposition of total hours worked $\log(H_t)$:

Two economies

Hansen (1985) considers two economies: a standard one-sector stochastic growth model (divisible labor) and a model with indivisible labor

The economy to be studied is populated by a continuum of identical infinitely lived households with names on the closed interval [0, 1]. There is a single firm with access to a technology described by a standard Cobb–Douglas production function of the form

$$f(\lambda_t, k_t, h_t) = \lambda_t k_t^{\theta} h_t^{1-\theta}, \tag{1}$$

where labor (h_t) and accumulated capital (k_t) are the inputs and λ_t is a random shock which follows a stochastic process to be described below. Agents

Production

Output, which is produced by the firm and sold to the households, can either be consumed (c_t) or invested (i_t), so the following constraint must be satisfied:

$$c_t + i_t \leq f(\lambda_t, k_t, h_t). \tag{2}$$

The law of motion for the capital stock is given by

$$k_{t+1} = (1-\delta)k_t + i_t, \qquad 0 \leq \delta \leq 1, \tag{3}$$

where δ is the rate of capital depreciation. The stock of capital is owned by the households who sell capital services to the firm.

The technology shock is assumed to follow a first-order Markov process. In particular, λ_t obeys the following law of motion:

$$\lambda_{t+1} = \gamma\lambda_t + \varepsilon_{t+1}, \tag{4}$$

Agent's problem

Notes

Economy II: Indivisible Labor

Hansen (1985): "Indivisibility of labor is modeled by restricting the consumption possibilities set so that individuals can either work full time, denoted by h_0, or not at all."

However, this will introduce non-convexity in the consumption possibilities set. In order to guarantee the solution Hansen (1985) requires individuals to choose lotteries rather than hours worked as in Rogerson (1984)

Hansen (1985): "Thus, each period, instead of choosing man-hours, households choose a probability of working, α_t.A lottery then determines whether or not the household actually works."

Agent's problem

Hansen (1985): "A key property of this economy is that the elasticity of substitution between leisure in different periods for the 'representative agent' is infinite."

Calibration

In order to calibrate this economy Hansen (1985) chose a distribution function, F, and specific parameter values for θ, δ, β, A, γ, and h_0:
"The parameter θ corresponds to capital's share in production. This has been calculated using U.S. time series data by Kydland and Prescott (1982,1984) and was found to be approximately 0.36.

The rate of depreciation of capital, δ, is set equal to 0.025 which implies an annual rate of depreciation of 10 percent.

The discount factor, β, is set equal to 0.99, which implies a steady state annual real rate of interest of four percent.

The parameter A in the utility function (5) is set equal to 2. This implies that hours worked in the steady state for the model with divisible labor is close to 1/3."

Notes

Results

Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

	Quarterly U.S. time series[a] (55.3–84.1)		Economy with divisible labor[b]		Economy with indivisible labor[b]	
Series	(a)	(b)	(a)	(b)	(a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)	1.76 (0.21)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)	0.51 (0.08)	0.87 (0.04)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)	5.71 (0.70)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)	0.47 (0.10)	0.05 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)	1.35 (0.16)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)	0.50 (0.07)	0.87 (0.03)

Results II

Hansen (1985): "When comparing the statistics describing the two artificial economies, one discovers that the economy with indivisible labor displays significantly larger fluctuations than the economy with divisible labor."

Conclusions

Hansen (1985): "In conclusion, this study demonstrates that non-convexities such as indivisible labor may be important for explaining the volatility of hours relative to productivity even when individuals are relatively unwilling to substitute leisure across time."

Notes