KYIV SCHOOL OF ECONOMICS

Financial Econometrics (2nd part): Introduction to Financial Time Series

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Please put your name below and answer ANY three out of four questions in 30 minutes:

If you attempt all four questions only **the first three** will be graded!

1. The Prime Minister of Dniprovia has requested you to evaluate the following AR(1)-GARCH(1,1) model fitted to the interest rate r_t on government bonds

$$\begin{aligned} r_t &= 0.05 - 0.01 r_{t-1} + a_t, a_t = \sigma_t \epsilon_t \\ \sigma_t^2 &= 0.05 + 0.01 a_{t-1}^2 + 0.95 \sigma_{t-1}^2, \end{aligned} \tag{1}$$

where all the coefficients are significant at 1% and ϵ_t follows a standard normal distribution.
a. Write the formulas for unconditional mean and unconditional variance and plug in the numbers. (Do not caclulate it - just write the numbers).

b. What should γ be equal to in $\sigma_t^2 = \gamma + 0.95\sigma_{t-1}^2$ to obtain an IGARCH(1,1) model without a drift?

c. Suppose that for a properly chosen m, the Ljung-Box $Q(m)$ statistics on a standardized shock $\widetilde{a_t}$ has a p-value of 0.78 and on $\widetilde{a_t}^2$ has a p-value of 0.00007. How would you compute $\widetilde{a_t}$?

How would you interpret these results? Explain in 1-2 sentences.

2. Write TRUE or FALSE and **explain** in 1 sentence.
a. When PACF and Akaike Information Criterion are used to identify the order p of the AR(p) model they produce the same result p.

b. In finite samples the sample autocorrelation coefficient $\hat{\rho}_l$ is an unbiased and consistent estimator of lag-l autocorrelation ρ_l.

c. The disadvantage of ARCH-GARCH models is the same effects of positive and negative shocks on volatility.

d. The threshold autoregressive model (TAR) may be stationary and ergodic even if the absolute value of the coefficient $\hat{\phi}_2$ on r_{t-2} term is greater than 1.

e. Suppose that in 2-state Makrov Switching Autoregressive (MSA) model of Hamilton (1989) the probability of moving from state 1 to state 2 is ω. Then the probability of moving from state 2 to state 1 is $1-\omega$.

3. Suppose that your research assistant estimated the following AR(2)-TAR-GARCH(1,1) model for the volatility

$$\begin{aligned} r_t &= 0.04 - 0.02 r_{t-2} + a_t, a_t = \sigma_t \epsilon_t \\ \sigma_t^2 &= 0.07 a_{t-1}^2 + 0.90 \sigma_{t-1}^2 \\ &+ I(a_{t-1} > 0)(0.05 - 0.05 a_{t-1}^2 - 0.06 \sigma_{t-1}^2), \end{aligned} \tag{2}$$

where all the coefficients are significant at 1% and ϵ_t follows a standard normal distribution.
a. What should be γ_1 and γ_2 in the following representation of TAR-GARCH(1,1) equation:

$$\sigma_t^2 = \begin{cases} \gamma_1 & \texttt{if} \quad a_{t-1} \leq 0 \\ \gamma_2 & \texttt{if} \quad a_{t-1} > 0? \end{cases}$$

b. Does the volatility respond symmetrically to positive and negative shocks? Explain carefully in 1-2 sentences.

c. What evidence would you use to check the adequacy of the model?

4. Perform the operations suggested below.

a. Apply the back-shift operator to $(1 - B^4)(1 - B)x_t$.

What is the likely periodicity of the data (week, month etc)?

b. Suppose that unemployment in the country of Poltavia can be described by the following AR(2) process:

$$u_t = 0.25 + 0.10u_{t-1} - 0.02u_{t-2} + a_t, \hat{\sigma}_a = 0.09. \tag{3}$$

Write the corresponding second-order difference equation.

Is unemployment in Poltavia characterized by business cycle behavior?

c. A 2-state Markov Switching Autoregressive model for GNP growth rate in Dniprovia is given by

$$g_t = \begin{cases} -0.5 + 0.5g_{t-1} + a_{1t} \texttt{ if state 1}, \\ 0.3 - 0.5g_{t-1} + a_{2t} \texttt{ if state 2} \end{cases}$$

where a_{1t} and a_{2t} are independent white noise series. Calculate the mean growth in each state.

Which state corresponds to expansion and which one to contraction?

d. Continue with the setup in Part c. Consider the probability of moving from state 1 to state 2 $p(st_t = 2|st_{t-1} = 1)$ and the probability of moving from state 2 to state 1 $p(st_t = 1|st_{t-1} = 2)$. What should be true about $p(st_t = 2|st_{t-1} = 1)$ and $p(st_t = 1|st_{t-1} = 2)$ if an expansion is more likely to lead to a contraction then vice versa?