KYIV SCHOOL OF ECONOMICS
Advanced Macroeconomics II, Fall 2013
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SUGGESTED SOLUTIONS TO HOMEWORK 2

I. Consider a standard infinite horizon problem with $U = \ln C_t$ and $Y_t = e^{z_t} \cdot K_t^{\alpha}$ where e is the base of the natural logarithm and z_t is productivity shock at time t. Suppose that the decision rule for capital is given by $K_{t+1} = \alpha\beta e^{z_t} \cdot K_t^{\alpha}$ where $0 < \beta < 1$ is the time preference parameter. Productivity shocks capture the business cycle component in the economy according to

$$z_{t+1} = \rho z_t + \sigma \cdot \epsilon_t \tag{1}$$

where ϵ_t is a random draw from the standard normal distribution. Suppose that in the country of Poltavia $\alpha = 0.4$, $\beta = 0.987$, $\rho = 0.95$ and $\sigma = 0.007$.
(i) Compute the steady state levels of K and C if $e^z = 1$ in the steady state.

$$K^* = (\alpha \cdot \beta)^{\frac{1}{1-\alpha}} \approx 0.213 \tag{2}$$
$$C^* = (K^*)^{\alpha} - K^* \approx 0.326 \tag{3}$$

(ii) Using any available software (Excel, MATLAB or similar) generate one time series of 100 shocks starting from $z_0 = 0$. Compute the time series for K and C if $K_0 = 0.1$. That is, obtain a value of z_1 and then compute K_1, Y_1 and C_1. Then obtain z_2, compute K_2 and so on.

Compute average values of K and C up to three decimal points (i.e. $K = 0.212$). Are average values of e^z, K and C equal to the steady state levels? Are they reasonably close? Explain in 1-2 sentences. Attach a graph depicting time periods on X-axis and time series for K and C on Y-axis.[1]

The average values are indeed very close but not equal to the steady state values. The errors come from the random process for shocks. Consider Figure 1 in Appendix.

(iii) Suppose that the government of Poltavia managed to fought corruption which led to an increase

[1] Observe that the average K and C will differ between iterations due to some randomness in process of productivity shocks. Ideally, one would repeat this process many times and then take averages but we skip this step for simplicity.

in α to 0.5. Explain in 2-3 sentences why α could have increased as a result of this action. Generate another time series of 100 shocks and compute new averages for K and C. Explain the difference in 2-3 sentences. Did consumers benefit?

It is possible that capital will be paid its *true* marginal product rather than an amount reduced by the level of bribes. For example, the MP_K of 0.5 can now include 0.1 paid as a bribe before. Although the steady state level of capital has increased consumers did not benefit because the steady state consumption has declined. This is not strange because higher interest rate induces consumers to invest more.

(iv) Suppose instead that people in Poltavia have learned about the upcoming epidemic of the flu which decreased β to $\frac{0.987}{2}$ in part (ii). Explain in 2-3 sentences why such news may lead to a decline in β. Generate another time series of 100 shocks and compute new averages for K and C. Explain the difference in 2-3 sentences. Did consumers benefit now?

Upcoming epidemic of the flu will reduce the probability of surviving till tomorrow. As a result, agents will place lower weight on tomorrow's consumption. Consumers will not benefit in this case as well because the consumption has declined.

(v) Let's think about the effects of the shock persistence parameter ρ. Suppose that the economy in part (ii) starts with $z_0 = 10$. Is this good or bad for agents? Would agents prefer to have higher or lower ρ in this case? Explain in 3-4 sentences.

It is actually good for agents because their consumption will sky-rocket. They would also prefer ρ to be as close to 1 as possible so that this high shock persists. However, notice that with a very low z_0 agents would prefer a very low persistence.

(vi) Agents typically do not have control over the volatility of productivity of shocks σ. However, agents care about the volatility of consumption C (measured by the standard deviation of C_t). Would agents be better or worse off if $\sigma = 0.7$ in part (ii)? Explain in 2-3 sentences.

Higher σ increases the volatility of the shock and, as a result, also increases the volatility of consumption. Thus, agents will be worse off with this higher σ.

II. Obtain equation 9 in Restuccia and Urrutia (2001). You can either follow the advice below or set up the problem using the infinite-horizon Lagrangian.

Use the budget constraint and the law of motion for capital on the bottom of page 103 to obtain

$$c = w(\kappa) + r(\kappa)k + \tau(\kappa) - (1+\theta)[(1+g)(1+n)k' - (1-\delta)k]. \tag{4}$$

Differentiate the Bellman with respect to k'. Take the envelope condition and update it one period ahead. Combine the first-order condition and the updated envelope condition to obtain the expression for $k^{\alpha-1}$:

$$k^{\alpha-1} = \frac{1+\theta}{\alpha}\left[\frac{(1+g)(1+n)}{\tilde{\beta}} - (1-\delta)\right] \tag{5}$$

Use the law of motion for capital to obtain the steady state condition for investment-to-capital ratio:

$$\frac{x}{k} = (1+g)(1+n) - (1-\delta). \tag{6}$$

Divide both sides by $k^{\alpha-1}$:

$$\frac{x}{k \cdot k^{\alpha-1}} = \frac{x}{k^{\alpha}} = \frac{x}{y} = \frac{(1+g)(1+n) - (1-\delta)}{k^{\alpha-1}}. \tag{7}$$

Now you can just plug in the expression for $k^{\alpha-1}$ on the right hand side to get equation (9).

III. Consider the model by Hansen (1985).

i. Set up the Lagrangian for the model with **divisible labor**. Obtain the Euler equation.

Given the Lagrangian:

$$L = E\sum_{t=0}^{\infty}\beta^t\big\{\ln c_t + A\ln l_t + \lambda_t(w_t[1-l_t] + r_t k_t - k_{t+1} + [1-\delta]k_t)\big\} \tag{8}$$

agents choose c_t, l_t and k_{t+1} every period t:

$$\begin{aligned} < c_t >: \frac{1}{c_t} &= \lambda_t && (9)\\ < l_t >: \frac{A}{l_t} &= \lambda_t w_t && (10)\\ < k_{t+1} >: \lambda_t &= E\beta\lambda_{t+1}[r_{t+1} + 1 - \delta] && (11)\\ & && (12) \end{aligned}$$

Once we replace λ's with marginal utilities we obtain the Euler equation

$$\frac{1}{c_t} = E\frac{\beta}{c_{t+1}}[r_{t+1} + 1 - \delta] \tag{13}$$

ii. Set up the Lagrangian for the model with **indivisible labor**. Obtain the Euler equation. Compare the two Euler equations.

Given the Lagrangian for the model for indivisible labor

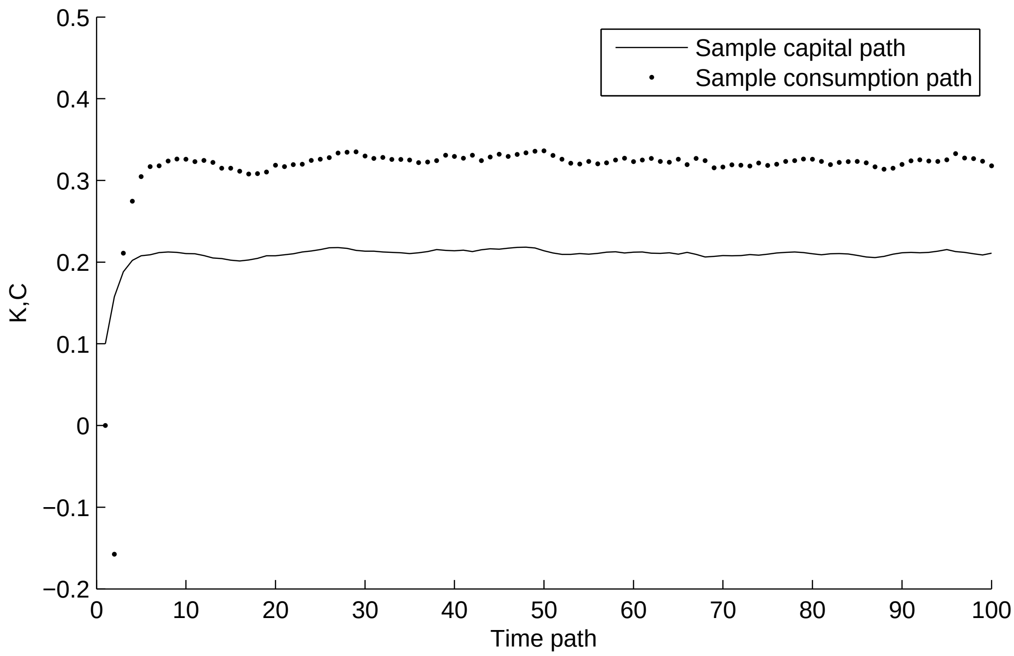
$$L = E\sum_{t=0}^{\infty} \beta^t \big\{ \ln c_t + \alpha_t A \ln(1 - h_0) + \lambda_t (w_t \alpha_t h_0 + r_t k_t - k_{t+1} + [1 - \delta]k_t) \big\} \tag{14}$$

agents choose c_t, α_t and k_{t+1} every period t. The first order conditions are the same except for condition for $< l_t >$ being replaced with a condition for α_t:

$$< \alpha_t >: A \ln(1 - h_0) = \lambda_t w_t h_0. \tag{15}$$

The Euler equation is the same as in the model with divisible labor.

Figure 1


0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
K,C
0
10
20
30
40
50
60
70
80
90
100
Time path
Sample capital path
Sample consumption path